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CONSTRUCTION OF BIPOLAR $\delta-$FUZZY *s*-EXTENSION AND ITS DECISION MAKING

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ABSTRACT: In this paper, the notion of bipolar fuzzy soft sets, the idea of a bipolar $\delta-$ fuzzy soft groupoid in a given set δ and related properties are investigated. Bipolar fuzzy soft groupoid is also constructed by using bipolar fuzzy soft groupoid. Conversely bipolar $\delta-$fuzzy soft groupoid is established by the way of bipolar fuzzy soft groupoid. The characterisations of bipolar $\delta-$fuzzy soft groupoid is provided and normal bipolar $\delta-$fuzzy soft groupoid are discussed. Finally the decision making approach for bipolar $\delta-$ fuzzy soft set is to be analysed with a suitable example.

Keywords: *Soft set, bipolar Fuzzy set, bipolar $\delta-$fuzzy soft groupoid, t-cut, s-cut, k-cut, Normal soft groupoid, s-fuzzy translation, s-extension, image.*

1. INTRODUCTION

In 1999, soft set theory was introduced by Molodtsov [11] as an alternative approach to fuzzy soft set theory, defined by Zadeh [12] in 1965. Soft ordered semi groups was defined by Jun et.al [6] , then a new structure called the soft inter- group was defined by Cagman et.al [4] and some properties of this new structure was obtained. A ring structure on soft sets was constructed by Acar et.al [2]. The soft inter-group was constructed by Kaygisiz [7] and the normal soft inter-group was defined and some properties were investigated. A traditional fuzzy set is characterized by the membership function, whose range is in the unit interval [0,1]. These are the several kinds of fuzzy extensions in the fuzzy set theory, for examples Intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets etc. Bipolar fuzzy set was introduced by Zhang [13] as a generalization of the fuzzy set. Bipolar fuzzy set is an extension of a fuzzy set whose membership degree interval is [-1,1]. Abdullah et.al [1] introduced the notion of bipolar fuzzy soft sets, combining soft sets and bipolar fuzzy soft sets and has also defined the operations of bipolar fuzzy soft sets. Kim et.al [9] has studied, the ideal theory of semi groups based on the bipolar valued fuzzy set. Fuzzy sub semi groups and fuzzy ideals, with operators in semi groups are defined by Hur et.al [5]. Therefore the notions of bipolar soft sets and their operations were defined by Koraaslam [10] and the concepts of bipolar fuzzy soft -semi group and bipolar fuzzy soft-ideals in semi groups.

2. Preliminaries and Basic Laws

In this section, we have discussed the basic idea and the elementary properties are explained.

Definition 2.1 Let U be a non empty finite set of objects called Universe and let E be a non empty parameters. An ordered pair (F, E) is said to be a soft set over U if F is a mapping from E into the set of all subsets of U. That is $F : E \to p(U)$

It has been interpreted that a soft set indeed is a parameterized family of subset of U.

Example 1: Let $U = \{x_1, x_2, x_3\}$ be the set of three phones and E= {size (y_1), colour (y_2), rate (y_3)} be the set of parameters where $A = \{y_1, y_2\} \subset E$.

Then $(F,A)=\{F(y_1)=\{x_1,x_2,x_3\},F(y_2)=\{x_1,x_3\}\}$ is the crisp soft set over U which describes the "attractiveness of the phones" which Mr. X (say) is going to buy.

Definition 2.2 Let G be the Universe of discourse, A bipolar fuzzy set A in G is an object having the form $A=\{(x,A_P(x),A_N(x))/x\in G\}$ where $A_P:G\to[0,1]$ and $A_N:G\to[-1,0]$ are mappings. The positive membership degree $A_P(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set A and the negative membership degree $A_N(x)$ denotes the satisfaction degree of x to some implicit counter property of A. It is possible for an element ' x 'to be $A_P(x)\neq 0$ and $A_N(x)\neq 0$ when the membership function of the property overlaps that of its counter property over some portion of the domain. By simplification, use the symbol $A=(G;A_P,A_N)$ where as the bipolar fuzzy set $A=\{(x,A_P(x),A_N(x))/x\in G\}$ by a groupoid of a group G, we mean the non empty subset B of G, such that $B^2\subseteq B$.

Definition 2.3 A fuzzy set μ in G is called a fuzzy groupoid of G, if it satisfies $\mu(xy)\geq\min\{\mu(x),\mu(y)\ \forall\ x,y\in G\}$. For any family $\{a_i/i\in\Omega\}$ of real numbers, we define

$$\max\{a_i/i\in\Omega\}=\begin{cases}\max\{a_i/i\in\Omega\} & \text{if } \Omega \text{ is finite}\\ \sup\{a_i/i\in\Omega\} & \text{otherwise}\end{cases}$$

$$\min\{a_i/i\in\Omega\}=\begin{cases}\min\{a_i/i\in\Omega\} & \text{if } \Omega \text{ is finite}\\ \inf\{a_i/i\in\Omega\} & \text{otherwise}\end{cases}$$

Definition 2.4 A bipolar fuzzy set $A=(G;A_P,A_N)$ in G is called a bipolar fuzzy soft groupoid G if it satisfies the following conditions $A_P(xy)\geq\min\{A_P(x),A_P(y)\}$ and $A_N(xy)\leq\max\{A_N(x),A_N(y)\}\ \forall\ x,y\in G$.

In what follows, Let G and δ denote a group and a non empty set respectively, unless otherwise specified.

Definition 2.5 A bipolar $\delta-$ fuzzy soft set (BPδ FSS) $A_\delta=<G\times\delta;A_P{}^\delta,A_N{}^\delta>$ in G is called a bipolar $\delta-$ fuzzy soft groupoid of G if it satisfies $A_P{}^\delta(xy,\alpha)\geq\min\{A_P{}^\delta(x,\alpha),A_P{}^\delta(y,\alpha)\}$ and $A_N{}^\delta(xy,\alpha)\leq\max\{A_N{}^\delta(x,\alpha),\ A_N{}^\delta(y,\alpha)\}\quad\forall\,x,y\in G \text{ and } \alpha\in\delta.$

Example 2: Consider a group $G=\{l,\ m\}$ with the following Cayley table

.	L	m
l	L	m
m	M	*l*

Let $\delta=\{1,2\}$ and let $A_\delta=\{G\times\delta;A_P{}^\delta,A_N{}^\delta\}$ be a bipolar $\delta-$ fuzzy soft set in G defined by
$A_\delta=\{<(l,1);-0.4,1>,<(l,2);-0.9,1>,<(m,1);-0.7,0.4>,<(m,2);-0.7,0.8>\}$.

It is easy to verify that $A_\delta=\{G\times\delta;A_P{}^\delta,A_N{}^\delta\}$ is a bipolar $\delta-$ fuzzy soft groupoid of G.

Basic Laws of bipolar $\delta-$ fuzzy soft sets

Proposition 2.1 (Demorgan's law): Let $A_\delta,B_\delta\in BP\delta FSS$.

Then prove that $(A_\delta\cap B_\delta)^C=A_\delta{}^C\cup B_\delta{}^C$

Proof: Let $x\in G$ and $\alpha\in\delta$

LHS:

$(A_\delta \cap B_\delta)^C(x,\alpha) \quad = 1-(A_\delta \cap B_\delta)(x,\alpha)$

$= 1-\max\{A_\delta(x,\alpha), B_\delta(x,\alpha)\}$

$= \min\{1-A_\delta(x,\alpha), 1-B_\delta(x,\alpha)\}$

$= \min\{A_\delta{}^C(x,\alpha), B_\delta{}^C(x,\alpha)\}$

RHS:

$(A_\delta{}^C \cup B_\delta{}^C)(x,\alpha) \quad = ((1-A_\delta)\cup(1-B_\delta))(x,\alpha)$

$= (1-A_\delta(x,\alpha))\cup(1-B_\delta(x,\alpha))$

$= \min\{1-A_\delta(x,\alpha), 1-B_\delta(x,\alpha)\}$

$= \min\{A_\delta{}^C(x,\alpha), 1-B_\delta{}^C(x,\alpha)\}$

$\therefore$ LHS = RHS

Proposition 2.2 (Distributive law): Let $A_\delta, B_\delta, C_\delta \in BP\delta\, FSS$. Then the following conditions are hold

(i) $A_\delta \cup (B_\delta \cap C_\delta) = (A_\delta \cup B_\delta) \cap (A_\delta \cup C_\delta)$

(ii) $A_\delta \cap (B_\delta \cup C_\delta) = (A_\delta \cap B_\delta) \cup (A_\delta \cap C_\delta)$

Proof: Let $x,\ y \in G$ and $\alpha \in \delta$

LHS:

$A_\delta \cup (B_\delta \cap C_\delta)(x,\alpha) \quad = \min\{A_\delta(x,\alpha), (B_\delta \cap C_\delta)(x,\alpha)\}$

$= \max\{\min\{A_\delta(x,\alpha), B_\delta(x,\alpha)\}, \min\{A_\delta(x,\alpha), C_\delta(x,\alpha)\}\}$

$= \max\{(A_\delta \cup B_\delta)(x,\alpha), (A_\delta \cup C_\delta)(x,\alpha)\}$

$= (A_\delta \cup B_\delta) \cap (A_\delta \cup C_\delta)(x,\alpha)$

RHS:

$(A_\delta \cup B_\delta) \cap (A_\delta \cup C_\delta)(x,\alpha) \quad = \max\{(A_\delta \cup B_\delta)(x,\alpha), (A_\delta \cup C_\delta)(x,\alpha)\}$

$= \max\{\min\{A_\delta(x,\alpha), B_\delta(x,\alpha)\}, \min\{A_\delta(x,\alpha), C_\delta(x,\alpha)\}\}$

$= \min\{\max\{A_\delta(x,\alpha), B_\delta(x,\alpha)\}, \max\{A_\delta(x,\alpha), C_\delta(x,\alpha)\}\}$

$= \min\{\max\{A_\delta(x,\alpha)\}, \{B_\delta(x,\alpha), C_\delta(x,\alpha)\}\}$

$= \min\{A_\delta(x,\alpha), \max\{B_\delta(x,\alpha), C_\delta(x,\alpha)\}\}$

$= A_\delta \cup (B_\delta \cap C_\delta)(x,\alpha)$

$\therefore$ LHS = RHS

Similarly we can show (ii).

Proposition 2.3 (Complementary Law): Let $A_\delta, B_\delta \in BP\delta\, FSS$. Then the following conditions are hold

(i) $(A_\delta \cup B_\delta)^C = A_\delta{}^C \cap B_\delta{}^C$

(ii) $(A_\delta \cap B_\delta)^C = A_\delta{}^C \cup B_\delta{}^C$

Proof: Let $x \in G$ and $\alpha \in \delta$

$(A_\delta \cup B_\delta)^C(x,\alpha) \quad = 1-(A_\delta \cup B_\delta)(x,\alpha)$

$= 1-\min\{A_\delta(x,\alpha), B_\delta(x,\alpha)\}$

$= \max\{1-A_\delta(x,\alpha), 1-B_\delta(x,\alpha)\}$

$= \max\{A_\delta{}^C(x,\alpha), B_\delta{}^C(x,\alpha)\}$

$(A_\delta{}^C \cap B_\delta{}^C)(x,\alpha) \quad = ((1-A_\delta)\cap(1-B_\delta))(x,\alpha)$

$$= \max\{1 - A_\delta(x,\alpha), 1 - B_\delta(x,\alpha)\}$$
$$= \max\{A_\delta^{\,C}(x,\alpha),\ B_\delta^{\,C}(x,\alpha)\}$$
$\therefore$ LHS = RHS

Similarly we can show (ii).

Proposition 2.4 (Associative Law): Let $A_\delta, B_\delta \in BP\delta\, FSS$. Then prove

(i) $A_\delta \cup (B_\delta \cup C_\delta) = (A_\delta \cup B_\delta) \cup C_\delta$

(ii) $A_\delta \cap (B_\delta \cap C_\delta) = (A_\delta \cap B_\delta) \cap C_\delta$

Proof:

$$(A_\delta \cup (B_\delta \cup C_\delta))(x,\alpha) = \min\{A_\delta(x,\alpha), (B_\delta \cup C_\delta)(x,\alpha)\}$$
$$= \min\{A_\delta(x,\alpha), \min\{B_\delta(x,\alpha), C_\delta(x,\alpha)\}\}$$
$$= \min\{\min\{A_\delta(x,\alpha), B_\delta(x,\alpha)\}, C_\delta(x,\alpha)\}$$
$$= \min\{(A_\delta \cup B_\delta)(x,\alpha), C_\delta(x,\alpha)\}$$
$$= (A_\delta \cup B_\delta)(x,\alpha) \cup C_\delta(x,\alpha)$$
$$= ((A_\delta \cup B_\delta) \cup C_\delta)(x,\alpha)$$
$$= (A_\delta \cup B_\delta) \cup C_\delta$$

Hence the proof. Similarly we can show (ii).

Theorem 2.1 Let δ be the set of all bipolar fuzzy soft groupoid of G and let $A_\delta = \{G \times \delta; A_P{}^\delta, A_N{}^\delta\}$ be a bipolar $\delta-$ fuzzy soft set in G when $A_N{}^\delta(x,A) = A_N(x)$ and $A_P{}^\delta(x,A) = A_P(x)$ for $x \in G$ and $A = (G; A_P, A_N)$. Then $A_\delta = \{G \times \delta; A_P{}^\delta, A_N{}^\delta\}$ is a bipolar $\delta-$ fuzzy soft groupoid of G.

Proof: Let $x, y \in G$ and $A = (G; A_P, A_N)$. Then

$A_P{}^\delta(xy, A) = A_P(xy) \ge \min\{A_P(x), A_P(y)\} = \min\{A_P{}^\delta(x), A_P{}^\delta(y)\}$ and

$A_N{}^\delta(xy, A) = A_N(xy) \le \max\{A_N(x), A_N(y)\} = \max\{A_N{}^\delta(x), A_N{}^\delta(y)\}$.

Hence $A_\delta = \{G \times \delta; A_P{}^\delta, A_N{}^\delta\}$ is a bipolar $\delta-$ fuzzy soft groupoid of G.

Theorem 2.2 If $A_\delta = \{G \times \delta; A_P{}^\delta, A_N{}^\delta\}$ is a bipolar $\delta-$ fuzzy soft groupoid of G and $\alpha \in \delta$, then a bipolar soft set $A = \{G; A_P{}^\alpha, A_N{}^\alpha\}$ where $A_P{}^\delta : G \to [0,1],\ x \to A_P{}^\delta(x,\alpha)$ and $A_N{}^\delta : G \to [-1,0],\ x \to A_N{}^\delta(x,\alpha)$ is a bipolar $\delta-$ fuzzy soft groupoid of G.

Proof: Let $x, y \in G$. Then

$A_P{}^\delta(xy,\alpha) = A_P{}^\delta(xy,\alpha) \ge \min\{A_P{}^\delta(x,\alpha), A_P{}^\delta(y,\alpha)\} = \min\{A_P{}^\alpha(x), A_P{}^\alpha(y)\}$ and

$A_N{}^\delta(xy,\alpha) = A_N{}^\delta(xy,\alpha) \le \max\{A_N{}^\delta(x,\alpha), A_N{}^\delta(y,\alpha)\} = \max\{A_N{}^\alpha(x), A_N{}^\alpha(y)\}$. This completes the proof.

Theorem 2.3 If $A = \{G; A_P{}^\alpha, A_N{}^\alpha\}$, $\alpha \in \delta$ is a bipolar fuzzy soft groupoid of G, then a bipolar fuzzy soft set $A_\delta = \{G \times \delta; A_P{}^\delta, A_N{}^\delta\}$ where $A_P{}^\delta : G \times \delta \to [0,1]$, $(x,\alpha) \to A_P{}^\alpha(x)$ and $A_N{}^\delta : G \times \delta \to [-1,0]$, $(x,\alpha) \to A_N{}^\alpha(x)$ is a bipolar $\delta-$ fuzzy soft groupoid of G.

Proof: For any $x, y \in G$, we have

$A_P{}^\delta(xy,\alpha) = A_P{}^\alpha(xy) \ge \min\{A_P{}^\alpha(x), A_P{}^\alpha(y)\} = \min\{A_P{}^\delta(x,\alpha), A_P{}^\delta(y,\alpha)\}$ and

$A_N{}^\delta(xy,\alpha) = A_N{}^\alpha(xy) \le \max\{A_N{}^\alpha(x), A_N{}^\alpha(y)\} = \min\{A_N{}^\delta(x,\alpha), A_N{}^\delta(y,\alpha)\}$.

Hence A_δ is a bipolar $\delta-$ fuzzy soft groupoid.

Theorem 2.4 Let $f_\delta = (G^\delta : f_P{}^\delta, f_N{}^\delta)$ be a bipolar fuzzy soft groupoid of G^δ and let $A_\delta = \{G \times \delta; A_P{}^\delta, A_N{}^\delta\}$ be a bipolar $\delta-$ fuzzy soft set in G define by

${A_P}^{\delta}(x,\alpha)=\max\left\{ {f_P}^{\delta}(u)/u\in G^{\delta},u(\alpha)=x\right\}$, ${A_N}^{\delta}(x,\alpha)=\min\left\{ {f_N}^{\delta}(u)/u\in G^{\delta}, u(\alpha)=x\right\}$ for all $x\in G$ and $\alpha\in\delta$. Then $A_{\delta}=\left\{G\times\delta; {A_P}^{\delta}, {A_N}^{\delta}\right\}$ is a bipolar $\delta-$ fuzzy soft groupoid in G.

Proof: Let $x, y\in G$ and $\alpha\in\delta$. Then

$$\begin{aligned}
{A_P}^{\delta}(xy,\alpha) &= \max\left\{ {f_P}^{\delta}(u)/u\in G^{\delta}, u(\alpha)=xy\right\}\\
&\geq \max\left\{ {f_P}^{\delta}(uv)/u, v\in G^{\delta}, u(\alpha)=x, v(\alpha)=y\right\}\\
&\geq \max\left\{\min\left\{ {f_P}^{\delta}(u), {f_P}^{\delta}(v)\right\}/u,v\in G^{\delta},u(\alpha)=x,v(\alpha)=y\right\}\\
&= \min\left\{ \max\left\{\{ {f_P}^{\delta}(u)/u\in G^{\delta}, u(\alpha)=x\}, \{ {f_P}^{\delta}(v)/v\in G^{\delta}, v(\alpha)=y\}\right\}\right\}\\
&= \min\left\{ {f_P}^{\delta}(x,\alpha), {f_P}^{\delta}(y,\alpha)\right\}
\end{aligned}$$

and

$$\begin{aligned}
{A_N}^{\delta}(xy,\alpha) &= \min\left\{ {f_N}^{\delta}(u)/u\in G^{\delta}, u(\alpha)=xy\right\}\\
&\leq \min\left\{ {f_N}^{\delta}(uv)/u, v\in G^{\delta}, u(\alpha)=x, v(\alpha)=y\right\}\\
&\leq \min\left\{\max\left\{ {f_N}^{\delta}(u), {f_N}^{\delta}(v)/u, v\in G^{\delta}, u(\alpha)=x, v(\alpha)=y\right\}\right\}\\
&= \max\left\{\min\left\{\left\{ {f_N}^{\delta}(u)/u\in G^{\delta}, u(\alpha)=x\right\}, \left\{ {f_N}^{\delta}(v)/v\in G^{\delta}, v(\alpha)=y\right\}\right\}\right\}\\
&= \max\left\{ {f_N}^{\delta}(x,\alpha), {f_N}^{\delta}(y,\alpha)\right\}
\end{aligned}$$

Hence A_{δ} is a bipolar $\delta-$ fuzzy soft groupoid of G.

Example 3: Let $G=\left\{l,m\right\}$ be a group in example (2) and let $\delta=\{1,2\}$. Then $G^{\delta}=\left\{e, a, b,c\right\}$ when $e(1)=e(2)=b(1)=b(2)=c(2)=l$ and $a(1)=a(2)=b(2)=c(2)=m$ is a group (a commutative group) under the following Cayley table

*	e	A	b	c
e	e	A	b	c
a	a	E	c	b
b	b	C	e	a
c	c	B	a	e

Let $f_{\delta}=(G^{\delta}; {f_P}^{\delta}, {f_N}^{\delta})$ be a bipolar fuzzy soft set in G^{δ} defined by

$f_{\delta}=\left\{(e; -0.7, 0.6), (a; -0.4, 0.2), (b; -0.9, 0.5), (c;-0.9,0.5)\right\}$. Then f_{δ} is a bipolar fuzzy soft groupoid of G^{δ}, Thus we can obtain a bipolar $\delta-$ fuzzy soft groupoid $A_{\delta}=\left\{G\times\delta; {A_P}^{\delta}, {A_N}^{\delta}\right\}$ of G as follows

$$\begin{aligned}
{A_P}^{\delta}(l,1) &= \max\left\{ {f_P}^{\delta}(u)/u\in G^{\delta},u(1)=l\right\}\\
&= \max\left\{ {f_P}^{\delta}(e), {f_P}^{\delta}(b)\right\}\\
&= 0.6\\
{A_P}^{\delta}(l,2) &= \max\left\{ {f_P}^{\delta}(u)/u\in G^{\delta},u(2)=l\right\}\\
&= \max\left\{ {f_P}^{\delta}(e), {f_P}^{\delta}(c)\right\}\\
&= 0.6\\
{A_P}^{\delta}(m,1) &= \max\left\{ {f_P}^{\delta}(u)/u\in G^{\delta},u(1)=m\right\}\\
&= \max\left\{ {f_P}^{\delta}(a), {f_P}^{\delta}(c)\right\}\\
&= 0.5\\
{A_P}^{\delta}(m,2) &= \max\left\{ {f_P}^{\delta}(u)/u\in G^{\delta},u(2)=m\right\}\\
&= \max\left\{ {f_P}^{\delta}(a), {f_P}^{\delta}(b)\right\}
\end{aligned}$$

$$
\begin{aligned}
& = 0.5 \\
A_N^{\delta}(l,1) & = \min\{f_N^{\delta}(v) / v \in G^{\delta}, v(1) = l\} \\
& = \min\{f_N^{\delta}(e), f_N^{\delta}(b)\} \\
& = -0.9 \\
A_N^{\delta}(l,2) & = \min\{f_N^{\delta}(v) / v \in G^{\delta}, v(2) = l\} \\
& = \min\{f_N^{\delta}(e), f_N^{\delta}(c)\} \\
& = -0.9 \\
A_N^{\delta}(m,1) & = \min\{f_N^{\delta}(v) / v \in G^{\delta}, v(1) = m\} \\
& = \min\{f_N^{\delta}(a), f_N^{\delta}(c)\} \\
& = -0.9 \\
A_N^{\delta}(m,2) & = \min\{f_N^{\delta}(v) / v \in G^{\delta}, v(2) = m\} \\
& = \min\{f_N^{\delta}(a), f_N^{\delta}(b)\} \\
& = -0.9
\end{aligned}
$$

Theorem 2.5 Let A_{δ} be a bipolar $\delta-$ fuzzy soft groupoid of G and let $f_{\delta} = (G^{\delta}; f_P^{\delta}, f_N^{\delta})$ be a bipolar fuzzy soft set in G^{δ} define by $f_P^{\delta}(u) = \min\{A_P^{\delta}(u(\alpha),\alpha) / \alpha \in \delta\}$ and $f_N^{\delta}(u) = \max\{A_N^{\delta}(u(\alpha),\alpha) / \alpha \in \delta\}$ for all $u \in G^{\delta}$. Then f_{δ} is a bipolar fuzzy soft groupoid of G^{δ}.

Proof: For any $u, v \in G^{\delta}$, we have

$$
\begin{aligned}
f_P^{\delta}(uv) & = \min\{A_P^{\delta}((uv)(\alpha),\alpha) / \alpha \in \delta\} \\
& = \min\{A_P^{\delta}(u(\alpha)v(\alpha),\alpha) / \alpha \in \delta\} \\
& \geq \min\{\min\{A_P^{\delta}(u(\alpha),\alpha) / \alpha \in \delta\}, \min\{A_P^{\delta}(v(\alpha),\alpha) / \alpha \in \delta\}\} \\
& = \min\{f_P^{\delta}(u), f_P^{\delta}(v)\} \quad \text{and} \\
f_N^{\delta}(uv) & = \max\{A_N^{\delta}((uv)(\alpha),\alpha) / \alpha \in \delta\} \\
& = \max\{A_N^{\delta}(u(\alpha)v(\alpha),\alpha) / \alpha \in \delta\} \\
& \leq \max\{\max\{A_N^{\delta}(u(\alpha),\alpha) / \alpha \in \delta\}, \max\{A_N^{\delta}(v(\alpha),\alpha) / \alpha \in \delta\}\} \\
& = \max\{f_N^{\delta}(u), f_N^{\delta}(v)\}
\end{aligned}
$$

Thus $f_{\delta} = (G^{\delta}; f_P^{\delta}, f_N^{\delta})$ be a bipolar fuzzy soft groupoid of G^{δ}.

Example 4: $A_{\delta} = \{G \times \delta; A_P^{\delta}, A_N^{\delta}\}$ be a bipolar $\delta-$ fuzzy soft groupoid. We assume G^{δ} is a commutative group in example (2) compare with example (3). Then we can induce a bipolar fuzzy soft groupoid for $(G^{\delta}; f_P^{\delta}, f_N^{\delta})$ of G^{δ} as follows

Positive	Negative
$f_P^{\delta}(e) = \min\{A_P^{\delta}(e(\alpha),\alpha) / \alpha \in \delta\}$ $= \min\{A_P^{\delta}(e(1),1), A_P^{\delta}(e(2),2)\}$ $= \min\{A_P^{\delta}(l,1), A_P^{\delta}(l,2)\}$ $= 0.6$ $f_P^{\delta}(a) = \min\{A_P^{\delta}(a(\alpha),\alpha) / \alpha \in \delta\}$ $= \min\{A_P^{\delta}(a(1),1), A_P^{\delta}(a(2),2)\}$ $= \min\{A_P^{\delta}(m,1), A_P^{\delta}(m,2)\}$ $= 0.3$	$f_N^{\delta}(e) = \max\{A_N^{\delta}(e(\alpha),\alpha) / \alpha \in \delta\}$ $= \max\{A_N^{\delta}(e(1),1), A_N^{\delta}(e(2),2)\}$ $= \max\{A_N^{\delta}(l,1), A_N^{\delta}(l,2)\}$ $= -0.9$ $f_N^{\delta}(a) = \max\{A_N^{\delta}(a(\alpha),\alpha) / \alpha \in \delta\}$ $= \max\{A_N^{\delta}(a(1),1), A_N^{\delta}(a(2),2)\}$ $= \max\{A_N^{\delta}(m,1), A_N^{\delta}(m,2)\}$

$f_P^{\delta}(b) = \min\{A_P^{\delta}(b(\alpha),\alpha)/\alpha\in\delta\}$ $= \min\{A_P^{\delta}(b(1),1), A_P^{\delta}(b(2),2)\}$ $= \min\{A_P^{\delta}(l,1), A_P^{\delta}(m,2)\}$ $= 0.5$ $f_P^{\delta}(c) = \min\{A_P^{\delta}(c(\alpha),\alpha)/\alpha\in\delta\}$ $= \min\{A_P^{\delta}(c(1),1), A_P^{\delta}(c(2),2)\}$ $= \min\{A_P^{\delta}(l,2), A_P^{\delta}(m,1)\}$ $= 0.3$	$= -0.9$ $f_N^{\delta}(b) = \max\{A_N^{\delta}(b(\alpha),\alpha)/\alpha\in\delta\}$ $= \max\{A_N^{\delta}(b(1),1), A_N^{\delta}(b(2),1)\}$ $= \max\{A_N^{\delta}(l,1), A_N^{\delta}(m,1)\}$ $= -0.9$ $f_N^{\delta}(c) = \max\{A_N^{\delta}(c(\alpha),\alpha)/\alpha\in\delta\}$ $= \max\{A_N^{\delta}(c(1),1), A_N^{\delta}(c(2),2)\}$ $= \max\{A_N^{\delta}(l,2), A_N^{\delta}(m,2)\}$ $= -0.9$

For a bipolar $\delta-$ fuzzy soft set A_δ in G and $(s,t)\in[-1,0]\times[0,1]$, we define

$P(A_\delta;t) = \{x\in G / A_P^{\delta}(x,\alpha)\geq t \quad \forall\ \alpha\in\delta\}$

$N(A_\delta;s) = \{x\in G / A_N^{\delta}(x,\alpha)\leq s \quad \forall\ \alpha\in\delta\}$

Which are called the positive t-cut of A_δ and negative s-cut of A_δ respectively. The set $D(A_\delta\ ;(s,t))=P(A_\delta\ ;t)\cap N(A_\delta\ ;s)$ is called (s,t)- cut of A_δ. For every $k\in[0,1]$, if $(s,t)=(-k,k)$, then the set $D(A_\delta\ ;k)= P(A_\delta\ ;k)\cap N(A_\delta\ ;-k)$ is called the k- cut of A_δ.

Theorem 2.6 Let a bipolar $\delta-$ fuzzy soft set A_δ in G is a bipolar $\delta-$fuzzy soft groupoid of G. Then the following assumptions are valid:

(i) $P(A_\delta;t)\neq \phi \Rightarrow P(A_\delta;t)$ is a soft groupoid of G $\quad \forall\ x\in[0,1]$

(ii) $N(A_\delta;s)\neq \phi \Rightarrow N(A_\delta;s)$ is a soft groupoid of G $\quad \forall\ x\in[-1,0]$

Proof:

(i) Now, let $t\in[0,1]$ be such that $P(A_\delta;t)\neq \phi$ if $x,y\in P(A_\delta;t)$, then $A_P^{\delta}(x,\alpha)\geq t$ and $A_P^{\delta}(y,\alpha)\geq t$ for $\alpha\in\delta$ and so $A_P^{\delta}(xy,\alpha)\geq \min\{A_P^{\delta}(x,\alpha),\ A_P^{\delta}(y,\alpha)\}\geq t$.
Hence $P(A_\delta;t)$ is a soft groupoid of G.

(ii) Let $s\in[-1,0]$ be such that $N(A_\delta;s)\neq \phi$ if $x,y\in N(A_\delta;s)$, then $A_N^{\delta}(x,\alpha)\leq s$ and $A_N^{\delta}(y,\alpha)\leq s$ for $\alpha\in\delta$. It follows that $A_N^{\delta}(xy,\alpha)\leq \max\{A_N^{\delta}(x,\alpha),\ A_N^{\delta}(y,\alpha)\}\leq s$.
Hence $N(A_\delta;s)$ is a soft groupoid of G.

Theorem 2.7 Let A_δ be a bipolar $\delta-$ fuzzy soft set in G satisfying two conditions (i) and (ii) in theorem 2.6. Then A_δ is a bipolar $\delta-$fuzzy soft groupoid of G.

Proof: Assume that A_δ is not a bipolar $\delta-$fuzzy soft groupoid of G. Then the condition is false. (ie) there exist $l,m\in G$ and $\alpha\in\delta$ such that

$A_N^{\delta}(lm,\alpha) > \max\{A_N^{\delta}(l,\alpha), A_N^{\delta}(m,\alpha)\}$ or $A_P^{\delta}(lm,\alpha) < \min\{A_P^{\delta}(l,\alpha), A_P^{\delta}(m,\alpha)\}$.

If $A_N^{\delta}(lm,\alpha) > \max\{A_N^{\delta}(l,\alpha), A_N^{\delta}(m,\alpha)\}$ then

$A_N^{\delta}(lm,\alpha) > s_\alpha \geq \max\{A_N^{\delta}(l,\alpha), A_N^{\delta}(m,\alpha)\}$ for some $s_\alpha\in[-1,0]$. It follows that $lm\in N(A_\delta;s_\alpha)$ but $lm\notin N(A_\delta;S_\alpha)$ which is a contradiction.

Therefore $A_N^{\delta}(xy,\alpha)\leq \max\{A_N^{\delta}(x,\alpha),\ A_N^{\delta}(y,\alpha)\}$ for all $x,y\in G$ and $\alpha\in\delta$. Now, if $A_P^{\delta}(lm,\alpha) < \min\{A_P^{\delta}(l,\alpha), A_P^{\delta}(m,\alpha)\}$, then $A_P^{\delta}(lm,\alpha) < t_\alpha \leq \min\{A_P^{\delta}(l,\alpha), A_P^{\delta}(m,\alpha)\}$

and so $lm \in P(A_\delta; t_\alpha)$ but $lm \notin P(A_\delta; t_\alpha)$. Thus $P(A_\delta; t_\alpha)$ is not a soft groupoid of G, which is a contradiction. Consequently, A_δ is a bipolar $\delta-$fuzzy soft groupoid of G.

3. Normal bipolar $\delta-$fuzzy soft groupoid

Definition 3.1 A bipolar $\delta-$fuzzy soft groupoid $A_\delta = \{G \times \delta; A_P{}^\delta, A_N{}^\delta\}$ of G is said to be a normal if it satisfies; $A_P{}^\delta(x,\alpha) = 1$ and $A_N{}^\delta(y,\alpha) = -1$ for all $x, y \in G$.

Definition 3.2 Let Δ denote the set of all normal bipolar $\delta-$fuzzy soft groupoids of G. Denote by ϕ the special element of s such that $A_P{}^\delta(\phi,\alpha) = \max_{x \in G} A_P{}^\delta(x,\alpha)$ and

$A_N{}^\delta(\phi,\alpha) = \min_{x \in G} A_N{}^\delta(x,\alpha)$ for all $\alpha \in \delta$. Clearly, if A_δ is a normal bipolar $\delta-$fuzzy soft groupoid of G, then $A_P{}^\delta(\phi,\alpha) = 1$ and $A_N{}^\delta(\phi,\alpha) = -1$ for all $\alpha \in \delta$. Further we consider a method for making a normal bipolar $\delta-$fuzzy soft groupoid from a given bipolar $\delta-$fuzzy soft groupoid.

Example 5: Consider a group $G^\delta = \{e, a, b, c\}$ which is described in example 3. Let $f_\delta = (G^\delta; f_P{}^\delta, f_N{}^\delta)$ be a a bipolar fuzzy soft set in G^δ defined by $f_\delta = \{(e;-0.7, 0.4), (a;-0.4, 0.2), (b;-0.6, 0.6), (e;-0.5, 0.8)\}$. Then f_δ is a bipolar fuzzy soft groupoid of G^δ, which induced a bipolar fuzzy soft groupoid f_δ, where

$A_P{}^\delta(l,1) = A_P{}^\delta(m,2) = 0.6 \cong 1$

$A_P{}^\delta(l,2) = A_P{}^\delta(m,1) = 0.8 \cong 1$

$A_N{}^\delta(l,1) = A_N{}^\delta(l,2) = -1$

$A_N{}^\delta(m,1) = A_N{}^\delta(m,2) = -1$

Theorem 3.1 Let $A_\delta = \{G \times \delta; A_P{}^\delta, A_N{}^\delta\}$ be a bipolar $\delta-$ fuzzy soft groupoid of G. Let $\overline{A}_\delta = \{G \times \delta; \overline{A}_P{}^\delta, \overline{A}_N{}^\delta\}$ be a bipolar fuzzy soft set in G defined by $A_P{}^\delta(x,\alpha) = A_P{}^\delta(x,\alpha) - A_P{}^\delta(\phi,\alpha) + 1$ and $A_N{}^\delta(x,\alpha) = A_N{}^\delta(x,\alpha) - A_N{}^\delta(\phi,\alpha) - 1$ for all $\alpha \in \delta$ and $x \in G$. Then $\overline{A}_\delta$ is a normal bipolar $\delta-$fuzzy soft groupoid of G.

Proof: For all $x, y \in G$ and $\alpha \in \delta$, We have

$$
\begin{aligned}
A_P{}^\delta(xy,\alpha) &= A_P{}^\delta(xy,\alpha) - A_P{}^\delta(\phi,\alpha) + 1 \\
&\geq \min\{A_P{}^\delta(x,\alpha),\ A_P{}^\delta(y,\alpha)\} - A_P{}^\delta(\phi,\alpha) + 1 \\
&= \min\{A_P{}^\delta(x,\alpha) - A_P{}^\delta(\phi,\alpha) + 1, A_P{}^\delta(y,\alpha) - A_P{}^\delta(\phi,\alpha) + 1\} \\
&= \min\{A_P{}^\delta(x,\alpha),\ A_P{}^\delta(y,\alpha)\} \\
A_N{}^\delta(xy,\alpha) &= A_N{}^\delta(xy,\alpha) - A_N{}^\delta(\phi,\alpha) - 1 \\
&\leq \max\{A_N{}^\delta(x,\alpha),\ A_N{}^\delta(y,\alpha)\} - A_N{}^\delta(\phi,\alpha) - 1 \\
&= \max\{A_N{}^\delta(x,\alpha) - A_N{}^\delta(\phi,\alpha) - 1, A_N{}^\delta(y,\alpha) - A_N{}^\delta(\phi,\alpha) - 1\} \\
&= \max\{A_N{}^\delta(x,\alpha),\ A_N{}^\delta(y,\alpha)\}
\end{aligned}
$$

$\therefore$ $\overline{A}_\delta$ is a normal bipolar $\delta-$fuzzy soft groupoid of G.

Definition 3.3 Let $\Omega: K \to H$ be a homomorphism of groups and let $S_\delta = \{H \times \delta; S_P{}^\delta, S_N{}^\delta\}$ be a bipolar $\delta-$fuzzy soft set in H. Then the inverse image of S_δ denoted by $\Omega^{-1}(S_\delta) = \{H \times \delta; \Omega^{-1}(S_P{}^\delta), \Omega^{-1}(S_N{}^\delta)\}$ is the bipolar $\delta-$fuzzy soft set in K given by

$\Omega^{-1}(S_P{}^{\delta})(x,\alpha)=(S_P{}^{\delta})(\Omega(x),\alpha)$ and $\Omega^{-1}(S_N{}^{\delta})(x,\alpha)=(S_N{}^{\delta})(\Omega(x),\alpha)$ for all $\alpha\in\delta$ and $x\in K$. Conversely, let A_δ be a bipolar $\delta-$fuzzy soft set in K. The image of A_δ written as $\Omega(A_\delta)=\{H\times\delta;\Omega(A_P{}^{\delta}),\ \Omega(A_N{}^{\delta})\}$ is a bipolar $\delta-$fuzzy soft set in H defined by

$$\Omega(A_P{}^{\delta})(y,\alpha)=\begin{cases}\max\limits_{z\in\Omega^{-1}(y)} A_P{}^{\delta}(z,\alpha) & if\ \ \Omega^{-1}(y)\neq\phi\\ 0 & otherwise\end{cases}$$ and

$$\Omega(A_N{}^{\delta})(y,\alpha)=\begin{cases}\min\limits_{z\in\Omega^{-1}(y)} A_N{}^{\delta}(z,\alpha) & if\ \ \Omega^{-1}(y)\neq\phi\\ 0 & otherwise\end{cases}$$ for all $\alpha\in\delta$ and $y\in H$,

where $\Omega^{-1}(y)=\{x/\Omega(x)=y\}$.

Theorem 3.2 Let $\Omega:K\to H$ be a homomorphism of groups and let S_δ be a bipolar $\delta-$fuzzy soft groupoid of H. Then its inverse image $\Omega^{-1}(S_\delta)$ is a bipolar $\delta-$fuzzy soft groupoid of K.

Proof: Let $x,y\in K$ and $\alpha\in\delta$

$$\begin{aligned}\Omega^{-1}(S_P{}^{\delta})(xy,\alpha)&=(S_P{}^{\delta})(\Omega(xy),\alpha)\\&=(S_P{}^{\delta})(\Omega(x)\Omega(y),\alpha)\qquad(\because\ \Omega\text{ is Homomorphism})\\&\geq\min\{S_P{}^{\delta}(\Omega(x),\alpha),\ S_P{}^{\delta}(\Omega(y),\alpha)\}\\&=\min\{\Omega^{-1}(S_P{}^{\delta})(x,\alpha),\ \Omega^{-1}(S_P{}^{\delta})(y,\alpha)\}\quad\text{and}\end{aligned}$$

$$\begin{aligned}\Omega^{-1}(S_N{}^{\delta})(xy,\alpha)&=(S_N{}^{\delta})(\Omega(xy),\alpha)\\&=(S_N{}^{\delta})(\Omega(x)\Omega(y),\alpha)\qquad(\because\ \Omega\text{ is Homomorphism})\\&\leq\max\{S_N{}^{\delta}(\Omega(x),\alpha),\ S_N{}^{\delta}(\Omega(y),\alpha)\}\\&=\max\{\Omega^{-1}(S_P{}^{\delta})(x,\alpha),\ \Omega^{-1}(S_N{}^{\delta})(y,\alpha)\}\end{aligned}$$

Hence $\Omega^{-1}(S_\delta)$ is a bipolar $\delta-$fuzzy soft groupoid of K.

Theorem 3.3 Let $\Omega:K\to H$ be a homomorphism between the groups K and H. If A_δ is a bipolar $\delta-$fuzzy soft groupoid of K, then the image $\Omega(A_\delta)$ is a bipolar $\delta-$fuzzy soft groupoid of H.

Proof: In this statement, we first show that

$\Omega^{-1}(y_1)\Omega^{-1}(y_2)\subseteq\Omega^{-1}(y_1\,y_2)$ for all $y_1,y_2\in H$ ⟶ (1)

For, if $x\in\Omega^{-1}(y_1)\Omega^{-1}(y_2)$ then $x=x_1x_2$ for some $x_1\in\Omega^{-1}(y_1)$ and $x_2\in\Omega^{-1}(y_2)$. Since Ω is a homomorphism, it follows that $\Omega(x)=\Omega(x_1\,x_2)=\Omega(x_1)\Omega(x_2)=y_1y_2$.

So that $x\in\Omega^{-1}(y_1\,y_2)$. Hence equation (1) holds

Now let $y_1,y_2\in H$ and $\alpha\in\delta$,

Assume that $y_1,y_2\notin I(\Omega)$, then $\Omega(A_P{}^{\delta})(y_1y_2,\alpha)=\Omega(A_N{}^{\delta})(y_1y_2,\alpha)=0$ but if $y_1,y_2\notin \mathrm{Im}(\Omega)$, then $\Omega^{-1}(y_1)=\phi$ (or) $\Omega^{-1}(y_2)\neq\phi$ by equation (1).

Thus $\Omega(A_P{}^{\delta})(y_1,\alpha)=\Omega(A_P{}^{\delta})(y_2,\alpha)=0$ or $\Omega(A_P{}^{\delta})(y_2,\alpha)=\Omega(A_P{}^{\delta})(y_2,\alpha)=0$ and

$\Omega(A_P{}^{\delta})(y_1y_2,\alpha)=\max\{\Omega(A_P{}^{\delta})(y_1,\alpha),\Omega(A_P{}^{\delta})(y_2,\alpha)\}=0$

$\Omega(A_N{}^{\delta})(y_1y_2,\alpha)=\min\{\Omega(A_N{}^{\delta})(y_1,\alpha),\Omega(A_N{}^{\delta})(y_2,\alpha)\}=0$

Suppose $\Omega^{-1}(y_1\,y_2)\neq\phi$, then we consider, two cases as follows

(i) $\Omega^{-1}(y_1)=\phi$ (or) $\Omega^{-1}(y_2)=\phi$

(ii) $\Omega^{-1}(y_1)=\phi$ and $\Omega^{-1}(y_2)\neq\phi$

Case (i) we have $\Omega(A_P{}^{\delta})(y_1,\alpha)=\Omega(A_N{}^{\delta})(y_1,\alpha)=0$, $\Omega(A_P{}^{\delta})(y_2,\alpha)=\Omega(A_N{}^{\delta})(y_2,\alpha)=0$.

Hence $\Omega(A_P{}^{\delta})(y_1y_2,\alpha)\geq \max\{\Omega(A_P{}^{\delta})(y_1,\alpha),\Omega(A_P{}^{\delta})(y_2,\alpha)\}$ and

$\Omega(A_N{}^{\delta})(y_1y_2,\alpha)\leq \min\{\Omega(A_N{}^{\delta})(y_1,\alpha),\Omega(A_N{}^{\delta})(y_2,\alpha)\}$

Case (ii)

$$\begin{aligned}
\Omega(A_P{}^{\delta})(y_1y_2,\alpha) &= \max_{z\in\Omega^{-1}(y_1y_2)} A_P{}^{\delta}(z,\alpha)\\
&\geq \max_{z\in\Omega^{-1}(y_1)\Omega^{-1}(y_2)} A_P{}^{\delta}(z,\alpha)\\
&= \max_{\substack{x_1\in\Omega^{-1}(y_1)\\ x_2\in\Omega^{-1}(y_2)}} A_P{}^{\delta}(x_1x_2,\alpha)\\
&\geq \max_{\substack{x_1\in\Omega^{-1}(y_1)\\ x_2\in\Omega^{-1}(y_2)}} \{\min\{A_P{}^{\delta}(x_1,\alpha),A_P{}^{\delta}(x_2,\alpha)\}\}\\
&= \min\left\{\max_{x_1\in\Omega^{-1}(y_1)} A_P{}^{\delta}(x_1,\alpha),\ \max_{x_2\in\Omega^{-1}(y_2)} A_P{}^{\delta}(x_2,\alpha)\right\}\\
&= \min\{\Omega(A_P{}^{\delta})(y_1,\alpha),\Omega(A_P{}^{\delta})(y_2,\alpha)\}
\end{aligned}$$

and

$$\begin{aligned}
\Omega(A_N{}^{\delta})(y_1y_2,\alpha) &= \min_{z\in\Omega^{-1}(y_1y_2)} A_N{}^{\delta}(z,\alpha)\\
&\leq \min_{Z\in\Omega^{-1}(y_1)\Omega^{-1}(y_2)} A_N{}^{\delta}(z,\alpha)\\
&= \min_{\substack{x_1\in\Omega^{-1}(y_1)\\ x_2\in\Omega^{-1}(y_2)}} A_N{}^{\delta}(x_1x_2,\alpha)\\
&\leq \min_{\substack{x\in\Omega^{-1}(y_1)\\ x\in\Omega^{-1}(y_2)}} \{\max\{A_N{}^{\delta}(x_1,\alpha),A_N{}^{\delta}(x_2,\alpha)\}\}\\
&= \max\left\{\min_{x_1\in\Omega^{-1}(y_1)} A_N{}^{\delta}(x_1,\alpha),\ \min_{x_2\in\Omega^{-1}(y_2)} A_N{}^{\delta}(x_2,\alpha)\right\}\\
&= \max\{\Omega(A_N{}^{\delta})(y_1,\alpha),\Omega(A_N{}^{\delta})(y_2,\alpha)\}
\end{aligned}$$

for all $y_1,y_2\in H$ and $\alpha\in\delta$.

4. Construction of $\delta-$fuzzy soft translation

In this section, we will discuss the basic idea of soft translation.

Definition 4.1 Let $A_{\delta}=(G\times\delta;A_P{}^{\delta},A_N{}^{\delta})$ be a bipolar $\delta-$fuzzy soft set of G and $(s,t)\in[-1,0]\times[0,1]$. By a bipolar $\delta-$fuzzy soft (s, t) translation of A_{δ}, we mean a bipolar $\delta-$fuzzy soft set $A_{\delta}{}^{(s,t)}=(G\times\delta;A_P{}^{\delta(t,T)},A_N{}^{\delta(s,T)})$ where

$A_{P^{\delta}}{}^{(t,T)}:G\times\delta\rightarrow[0,1]$ is a mapping defined by $A_{P^{\delta}}{}^{(t,T)}(x)=A_P{}^{\delta}(x,\alpha)+t\ \forall\ x\in G,\ \alpha\in G$

and $A_{N^{\delta}}{}^{(s,T)}:G\times\delta\rightarrow[-1,0]$ is a mapping defined by

$A_{N^{\delta}}{}^{(s,T)}(x)=A_N{}^{\delta}(x,\alpha)+s\quad \forall\ x\in G,\ \alpha\in G$.

Definition 4.2 A bipolar $\delta-$fuzzy soft groupoid A_{δ} of a groupoid of G is called a bipolar $\delta-$fuzzy soft bi-ideal of G if

(i) $A_P{}^{\delta}(xyz)\geq\min\{A_P{}^{\delta}(x),A_P{}^{\delta}(z)\}$

(ii) $A_N^{\delta}(xyz) \le \max\{A_N^{\delta}(x), A_N^{\delta}(z)\}$ for all $x, y, z \in G$.

Example 6: In example (3) we defined bipolar $\delta-$fuzzy soft set $A_{\delta} = (G\times\delta; A_P^{\delta}, A_N^{\delta})$ of G as follows

G^{δ}	e	A	b	c
A_P^{δ}	0.6	0.2	0.5	0.3
A_N^{δ}	-0.7	-0.4	-0.9	-0.9

Let $s = -0.1$ and t = 0.4. Then the bipolar $\delta-$fuzzy soft (s, t) translation $A_{(s,t)}^{\delta T} = (G\times\delta; A_P^{\delta}(s,T), A_N^{\delta}(t,T))$ of $A_{\delta} = (G\times\delta; A_P^{\delta}, A_N^{\delta})$ is

G^{δ}	e	A	b	C
$A_P^{\delta}(s,T)$	0.9	0.6	0.9	0.7
$A_N^{\delta}(s,T)$	-0.8	-0.5	-1.0	-1.0

Theorem 4.1 Let $A_{\delta} = (G\times\delta; A_P^{\delta}, A_N^{\delta})$ be a non-empty bipolar $\delta-$fuzzy soft subset of G and $(s,t) \in [-1,0]\times[0,1]$. Then the bipolar $\delta-$fuzzy soft translation A_{δ}^{T} is a bipolar $\delta-$fuzzy soft groupoid of G if and only if A_{δ} is a bipolar $\delta-$fuzzy soft groupoid of G.

Proof: Let A_{δ} be a bipolar fuzzy soft groupoid of G and $x, y \in G^{\delta}$. Then

$$\begin{aligned} A_{P^{\delta}}^{(t,T)}(xy) &= A_P^{\delta}(xy,\alpha) + t \\ &\ge \min\{A_P^{\delta}(x,\alpha), A_P^{\delta}(y,\alpha)\} + t \\ &= \min\{A_P^{\delta}(x,\alpha)+t, A_P^{\delta}(y,\alpha)+t\} \\ &= \min\{A_{P^{\delta}}^{(t,T)}(x), A_{P^{\delta}}^{(t,T)}(y)\} \end{aligned}$$

$$\begin{aligned} A_{N^{\delta}}^{(s,T)}(xy) &= A_N^{\delta}(xy,\alpha) + s \\ &\le \max\{A_N^{\delta}(x,\alpha), A_N^{\delta}(y,\alpha)\} + s \\ &= \max\{A_N^{\delta}(x,\alpha)+s, A_N^{\delta}(y,\alpha)+s\} \\ &= \max\{A_{N^{\delta}}^{(s,T)}(x), A_{N^{\delta}}^{(s,T)}(y)\} \end{aligned}$$

Hence $A_{\delta}^{T}(s,t)$ is a bipolar $\delta-$fuzzy soft groupoid of G.

Conversely, let $A_{\delta}^{T}(s,t)$ be a bipolar $\delta-$fuzzy soft groupoid of G for some $(s,t) \in [-1,0]\times[0,1]$. Then for any $x, y \in G^{\delta}$ we have

$$\begin{aligned} A_P^{\delta}(xy,\alpha)+t &= A_{P^{\delta}}^{(t,T)}(xy) \\ &\ge \min\{A_{P^{\delta}}^{(t,T)}(x), A_{P^{\delta}}^{(t,T)}(y)\} \\ &= \min\{A_P^{\delta}(x,\alpha)+t, A_P^{\delta}(y,\alpha)+t\} \\ &= \min\{A_P^{\delta}(x), A_P^{\delta}(y)\} \text{ and} \end{aligned}$$

$$\begin{aligned} A_N^{\delta}(xy,\alpha)+s &= A_{N^{\delta}}^{(s,T)}(xy) \\ &\le \max\{A_{N^{\delta}}^{(s,T)}(x), A_{N^{\delta}}^{(s,T)}(y)\} \\ &= \max\{A_N^{\delta}(x,\alpha)+s, A_N^{\delta}(y,\alpha)+s\} \end{aligned}$$

$$= \max\left\{A_N{}^{\delta}(x), A_N{}^{\delta}(y)\right\}$$

which implies that A_δ is a bipolar $\delta-$fuzzy soft groupoid of G.

Definition 4.3 Let $A_\delta = (G\times\delta; A_P{}^{\delta}, A_N{}^{\delta})$ and $A_\Delta = (G\times\Delta; A_P{}^{\Delta}, A_N{}^{\Delta})$ be two bipolar $\delta-$fuzzy soft sets of G^δ. If $A_P{}^{\delta}(x,\alpha) \le A_P{}^{\Delta}(x,\alpha)$ and $A_N{}^{\delta}(x,\alpha) \ge A_N{}^{\Delta}(x,\alpha)$ for all $x \in G^\delta$ and $\alpha \in \delta$. Then we say that A_Δ is bipolar $\delta-$fuzzy soft extension of A_δ.

Example 7: Let $A_\Delta = (G\times\Delta; A_P{}^{\Delta}, A_N{}^{\Delta})$ be a bipolar $\delta-$fuzzy soft set of a groupoid of G^δ in example (6) and it is defined as follows.

G^δ	E	A	b	c
$A_P{}^{\Delta}$	0.63	0.62	0.72	0.75
$A_N{}^{\Delta}$	-0.83	-0.52	-0.92	-0.92

Then A_Δ is a bipolar $\delta-$fuzzy soft extension of A_δ.

5. Bipolar $\delta-$fuzzy s-extension of soft sub groupoid

Definition 5.1 let A_Δ and A_δ be two bipolar $\delta-$fuzzy soft sets. Then A_Δ is called bipolar $\delta-$fuzzy $s-$extension of A_δ if the following hold

(i) A_Δ is bipolar $\delta-$fuzzy extension of A_δ.

(ii) A_δ is a bipolar $\delta-$fuzzy soft groupoid of G^δ.

Definition 5.2 The union of any two bipolar $\delta-$fuzzy soft sets A_Δ and A_δ is a bipolar $\delta-$fuzzy soft set $(A_\delta \cup A_\Delta)^\delta(x,\alpha) = \min\{A_\delta(x,\alpha), A_\Delta(x,\alpha)\}$ where $(A_\delta \cup A_\Delta)^\delta : G\times\delta \to [0,1]$ and $(A_\delta \cap A_\Delta)(x,\alpha) = \max\{A_\delta(x,\alpha), A_\Delta(x,\alpha)\}$ where $(A_\delta \cap A_\Delta)^\delta : G\times\delta \to [-1,0]$ for all $x \in G^\delta$ and $\alpha \in \delta$.

Theorem 5.1 Union of two bipolar $\delta-$fuzzy soft $s-$extension of a bipolar $\delta-$fuzzy soft groupoid A_δ in G^δ is a bipolar $\delta-$fuzzy soft $s-$extension of A_δ in G^δ.

Proof :

Let $A_\Delta = (G\times\delta; A_P{}^{\Delta}, A_N{}^{\Delta})$ and $A_\pi = (G\times\delta; A_P{}^{\pi}, A_N{}^{\pi})$ be two bipolar $\delta-$fuzzy soft $s-$extension of A_δ in G^δ. Then $A_P{}^{\delta}(x,\alpha) \le A_P{}^{\Delta}(x,\alpha)$, $A_P{}^{\delta}(x,\alpha) \le A_P{}^{\pi}(x,\alpha)$ and $A_N{}^{\delta}(x,\alpha) \ge A_N{}^{\Delta}(x,\alpha)$, $A_N{}^{\delta}(x,\alpha) \ge A_N{}^{\pi}(x,\alpha)$ for all $x \in G^\delta$ and $\alpha \in \delta$. Now

$(A_\Delta \cup A_\pi)_P(x,\alpha) = \max\{A_P{}^{\Delta}(x,\alpha), A_P{}^{\pi}(x,\alpha)\} \ge A_P{}^{\delta}(x,\alpha)$ and

$(A_\Delta \cup A_\pi)_N(x,\alpha) = \min\{A_N{}^{\Delta}(x,\alpha), A_N{}^{\pi}(x,\alpha)\} \le A_N{}^{\delta}(x,\alpha)$

Consequently, let $A_\Delta \cup A_\pi = (G\times\delta; (A_\Delta \cup A_\pi)_P{}^{\delta}, (A_\Delta \cup A_\pi)_N{}^{\delta})$ is a bipolar $\delta-$fuzzy soft $s-$extension of A_δ. Since A_δ is a bipolar $\delta-$fuzzy soft groupoid of G^δ, So $A_\Delta \cup A_\pi$ is a bipolar $\delta-$fuzzy soft $s-$extension of A_δ.

Theorem 5.2 Intersection of two bipolar $\delta-$fuzzy soft $s-$extension of a bipolar $\delta-$fuzzy soft groupoid A_δ in G^δ is a bipolar $\delta-$fuzzy soft $s-$extension of A_δ in G^δ.

Proof:

Let $A_\Delta = (G\times\delta; A_P{}^{\Delta}, A_N{}^{\Delta})$ and $A_\pi = (G\times\delta; A_P{}^{\pi}, A_N{}^{\pi})$ be two bipolar $\delta-$fuzzy soft $s-$extension of A_δ in G^δ. Then

$A_P{}^{\delta}(x,\alpha) \geq A_P{}^{\Delta}(x,\alpha)$, $A_P{}^{\delta}(x,\alpha) \geq A_P{}^{\pi}(x,\alpha)$, and
$A_N{}^{\delta}(x,\alpha) \leq A_N{}^{\Delta}(x,\alpha)$, $A_N{}^{\Delta}(x,\alpha) \leq A_N{}^{\pi}(x,\alpha)$ for all $x \in G^{\delta}$ and $\alpha \in \delta$.
Now
$(A_{\Delta} \cap A_{\pi})_P(x,\alpha) = \min\{A_P{}^{\Delta}(x,\alpha),\ A_P{}^{\pi}(x,\alpha)\} \leq A_P{}^{\delta}(x,\alpha)$ and
$(A_{\Delta} \cap A_{\pi})_N(x,\alpha) = \max\{A_N{}^{\Delta}(x,\alpha),\ A_N{}^{\pi}(x,\alpha)\} \geq A_N{}^{\delta}(x,\alpha)$
Consequently, let $A_{\Delta} \cap A_{\pi} = (G\times\delta;\ (A_{\Delta} \cap A_{\pi})_P{}^{\delta}, (A_{\Delta} \cap A_{\pi})_N{}^{\delta})$ is a bipolar $\delta-$fuzzy soft $s-$extension of A_{δ}. Since A_{δ} is a bipolar $\delta-$fuzzy soft groupoid of G^{δ}, So $A_{\Delta} \cap A_{\pi}$ is a bipolar $\delta-$fuzzy soft $s-$extension of A_{δ} .

Example 8: A_{Δ} is example (7) in a bipolar $\delta-$fuzzy soft $s-$extension of A_{δ} in example (6). Let $A_{\delta} = (G\times\delta; A_P{}^{\delta}, A_N{}^{\delta})$ be $BP\delta FSS$. Then for all $x, y \in G^{\delta}$ and $\alpha \in \delta$, We have bipolar $\delta-$fuzzy soft (s,t)- translation is defined by $A_{P^{\delta}}{}^{(t,T)}(x,\alpha) \geq A_P{}^{\delta}(x,\alpha)$ and
$A_{N^{\delta}}{}^{(s,T)}(x,\alpha) \leq A_N{}^{\delta}(x,\alpha)$ all $x, y \in G^{\delta}$ and $\alpha \in \delta$.

Remark:

Let A_{δ} be a bipolar $\delta-$fuzzy soft groupoid of G^{δ} and $(s,t) \in [-1,0]\times[0,1]$. Then bipolar $\delta-$fuzzy soft (s, t)-translation $A^T{}_{(s,T)}$ is a bipolar $\delta-$fuzzy $s-$extension of A_{δ} . The converse of the above statement is not in general. Infact A_{Δ} in example (7) is a bipolar $\delta-$fuzzy $s-$extension of A_{δ} of example (6) but it is not a bipolar $\delta-$fuzzy soft (s, t)-translation of A_{δ} in G^{δ} .

6. Decision making approach for bipolar $\delta-$fuzzy soft set

Algorithm:

Step (1) Construct D_1 complement and D_2 complement for the decision matrix
Step (2) Calculate $BF\delta(D_1{}^C, D_2{}^C)$ and $BF\delta(D_2{}^C, D_1{}^C)$
Step (3) Calculate score function
Step (4) Calculate the correlation coefficient between diseases and symptoms
Step (5) Choose maximum value
Step (6) Choose rank the order

Step (1): Numerical example we construct the following bipolar value problem based on the above algorithm

Bipolar $\delta-$fuzzy soft set D_1

D_1 =

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[-0.2, 0.5]	[-0.1, 0.6]	[-0.3, 0.7]	[-0.2, 0.6]
α_2	[-0.1, 0.4]	[-0.2, 0.5]	[-0.2, 0.6]	[-0.1, 0.5]
α_3	[-0.3, 0.4]	[-0.1, 0.4]	[-0.1, 0.5]	[-0.1, 0.4]

Bipolar $\delta-$fuzzy soft set D_2

D_2 =

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer

α_1	[-0.1, 0.3]	[-0.3, 0.4]	[-0.1, 0.3]	[-0.2, 0.5]
α_2	[-0.2, 0.4]	[-0.1, 0.5]	[-0.3, 0.4]	[-0.1, 0.4]
α_3	[-0.3, 0.5]	[-0.2, 0.3]	[-0.1, 0.2]	[-0.3, 0.2]

Bipolar $\delta-$fuzzy soft set D_1 Complement

$D_1^C =$

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[-0.8, 0.5]	[-0.9, 0.4]	[-0.7, 0.3]	[-0.8, 0.4]
α_2	[-0.9, 0.6]	[-0.8, 0.5]	[-0.8, 0.4]	[-0.9, 0.5]
α_3	[-0.7, 0.6]	[-0.9, 0.6]	[-0.9, 0.5]	[-0.9, 0.6]

Bipolar $\delta-$fuzzy soft set D_2 Complement

$D_2^C =$

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[-0.9, 0.7]	[-0.7, 0.6]	[-0.9, 0.7]	[-0.8, 0.5]
α_2	[-0.8, 0.6]	[-0.9, 0.5]	[-0.7, 0.6]	[-0.9, 0.6]
α_3	[-0.7, 0.5]	[-0.8, 0.7]	[-0.9, 0.8]	[-0.7, 0.8]

Step (2) Calculate $BF\delta(D_1^C, D_2^C)$

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
$\alpha_1\ \alpha_2$	[-0.8, 0.5]	[-0.8, 0.4]	[-0.7, 0.3]	[-0.8, 0.4]
$\alpha_2\ \alpha_3$	[-0.7, 0.6]	[-0.8, 0.5]	[-0.8, 0.4]	[-0.9, 0.5]
$\alpha_1\ \alpha_3$	[-0.7, 0.5]	[-0.9, 0.4]	[-0.7, 0.3]	[-0.8, 0.4]

Calculate $BF\delta(D_2^C, D_1^C)$

Symptoms	Diseases			
	Fever	Headache	Typhoid	Cancer
α_1	[-0.8, 0.7]	[-0.7, 0.5]	[-0.7, 0.6]	[-0.8, 0.5]
α_2	[-0.7, 0.5]	[-0.8, 0.5]	[-0.7, 0.6]	[-0.7, 0.6]
α_3	[-0.7, 0.5]	[-0.7, 0.6]	[-0.9, 0.7]	[-0.8, 0.5]

Step (3): Calculate score function $A_P^{\delta} = \sum_{i=1}^{n}\left(\frac{a+b}{3}\right)$ and $A_N^{\delta} = \sum_{j=1}^{n}\left(\frac{a+b}{3}\right)$

	Diseases

Symptoms	Fever	Headache	Typhoid	Cancer
α_1	[-0.533, 0.400]	[-0.530, 0.330]	[-0.466, 0.300]	[-0.330, 0.300]
α_2	[-0.466, 0.366]	[-0.533 0.330]	[-0.500, 0.330]	[-0.530, 0.360]
α_3	[-0.466, 0.333]	[-0.533, 0.330]	[-0.533, 0.330]	[-0.533, 0.330]

Score function $S = \frac{c+d}{2} = \frac{-5.959+4.012}{2} = -0.9735$

where c is the sum of negative value and d is the sum of positive value.

Step (4): Calculate correlation coefficient between diseases and symptoms by using the formula is given by

$$\rho = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}(d_{ij})^2}{\sqrt{\sum_{i=1}^{n}(d_i)^2}\sqrt{\sum_{j=1}^{n}(d_j)^2}}$$

Using step(3) we form a new calculation table as follows

Y / X	1	2	3	4
α_1	-0.13	-0.230	-0.166	-0.030
α_2	-0.10	-0.203	-0.170	-0.170
α_3	-0.13	-0.203	-0.203	-0.203

$$\rho = 0.3108 < 1$$

Step (5): Score function = - 0.9735 = - 0.97

Correlation function = 0.31

$\therefore$ Maximum value = 0.31

Step (6): From the table the order preference is given from step (4) as given below

Rank the order $r_4 < r_1 < r_3 < r_2$

$r_1 < r_3 < r_4 < r_2$

$r_1 < r_2 < r_3 < r_4$

For α_1 symptoms Cancer < Fever < Typhoid < Headache

For α_2 symptoms Fever < Typhoid < Cancer < Headache

For α_3 symptoms Fever < Headache < Typhoid < Cancer

Result:

Finally we conclude that symptoms (3) (ie) α_3 which is decide to choose the nearest ranking order for the given problem.

Conclusion:

The concepts of bipolar $\delta-$fuzzy soft groupoid have been discussed in groups by using a set δ. A bipolar $\delta-$fuzzy soft groupoid has been defined by using a bipolar fuzzy soft groupoid and vice versa. It has been stated about how the homomorphic images and inverse images of bipolar $\delta-$fuzzy soft groupoid become bipolar $\delta-$ fuzzy soft groupoid. Some properties are described using bipolar $\delta-$fuzzy soft normal groupoid. Decision making problems have to be defined by using bipolar $\delta-$ fuzzy soft sets.

SCOPE FOR FUTURE RESEARCH:

In future, the bipolar $\delta-$fuzzy soft groupoid with operators of various ideal structures can be studied by anyone based on the group theory.

References:

[1] S.Abdullah, M.Aslam and K.Ullah, Bipolar fuzzy soft sets and its application in decision making problem journal of intelligent and fuzzy systems 27(2),(2014) 729-742.

[2] U.Acar, F.Koyuncu and B.Tanay soft sets and soft rings computer and mathematics with application 59 (2010) 3458-3463.

[3] M.Akram, J.Kavikumar and A.B.Khamis, characterisation of Bipolar fuzzy soft semi groups Indian journal of science and technology 7(A) (2014),1211-1221

[4] N.Cagman , F.Citak and H.Aktas soft int-group, neural computing applications, 21(1), (2012), 151-158.

[5] K.Hur, Y.B.Jun and H.S.Kim, fuzzy sub semi groups and fuzzy ideals with operators in semi groups, Ann fuzzy math inform,1(2011),1-12

[6] JunY.B,K.J.Lee and j.zhen soft p-ideals of soft BCI algebras, computer mathematics with applications 58(2009),2060-2068.

[7] K.Kaygisiz, on soft int-groups, Annals of fuzzy mathematics and informatics, 4(2), (2012), 363-375.

[8] K.Kaygisiz, normal soft int-groups, anxiv, 1209,315-721(math, GR),2012.

[9] C.S.Kim.J.G.Kang and J.M.Kang, ideal theory of sub semi groups based on the bipolar valued fuzzy set theory, Ann.Fussy math inform 2(2011),103-206.

[10] P.Koraaslan and S.Korotas, a new approach bipolar soft sets and its applications, discrete mathematics, Algorithms and applications 7(4),(2015).

[11] D.A.Molodtsov, soft set theory –first results, computer and mathematics with applications 37(1999),19-31.

[12] A.Zadeh, fuzzy sets, information control 8(1965), 338-353.

[13] W.R.Zhang, Bipolar fuzzy sets, processing of fuzzy-IEEE 1994, PP, P35-840.

[14] Naim Cagman , Filiz citak and serdar Enginoglu , " Fuzzy parameterized Fuzzy soft set theory and its application " Turkish journal of Fuzzy systems Vol 1, No, 1 pp 21-35, 2010.

[15] *M.Dagdeviren, A hybrid multi criteria decision making model for personnel selection in manufacturing system. Journal of intelligent manufacturing, 21 (2010), 451 -460.*

[16] Tchangani, A., Bouzarour-Amokrane, Y. & peres, F(2012).Evaluation model in decision analysis; Bipolar approach, Informatica, 23(3), 461-485.

[17] Richard E. Bellman and Lotfi A. Zadeh. Decision - making in a fuzzy environment, Management Science, 17: 141- 164, 1970.